Topic 6: Understand Relationships Involving Triangles

Term	Meaning	Example
Perimeter		
Right Triangle		
Legs		
Hypotenuse		
Pythagorean Theorem		
Proof		
Converse of the Pythagorean Theorem		
Triangle Inequality Theorem		
Vertical Angles		

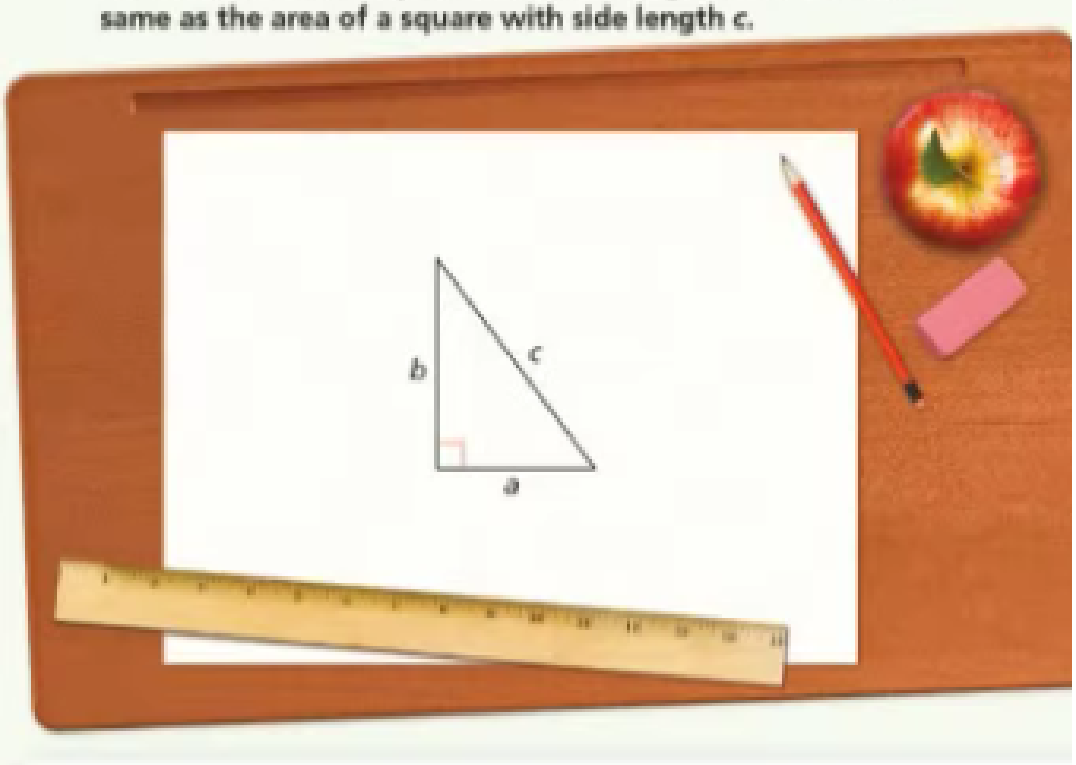
Adjacent Angles		
Complementary Angles		
Supplementary Angles		
Triangle Sum Theorem		
Remote Interior Angles		
Exterior Angle of a Triangle		
Polygon		
Regular Polygon		
Polygon Angle Sum Formula		

Lesson 1: Understand the Pythagorean Theorem

*Goal: Understand a **proof** of the **Pythagorean Theorem***
*Use the Pythagorean Theorem to find the **hypotenuse or leg** of a right triangle*

Explain It!

Kelly drew a right triangle on graph paper. Kelly says that the sum of the areas of squares with side lengths a and b is the same as the area of a square with side length c.



A. Do you agree with Kelly? Explain.

B. Sam drew a different right triangle with side lengths $a = 5$, $b = 12$, and $c = 13$. Is the relationship Kelly described true for Sam's right triangle? Explain.

EXAMPLE 1 Understand the Pythagorean Theorem

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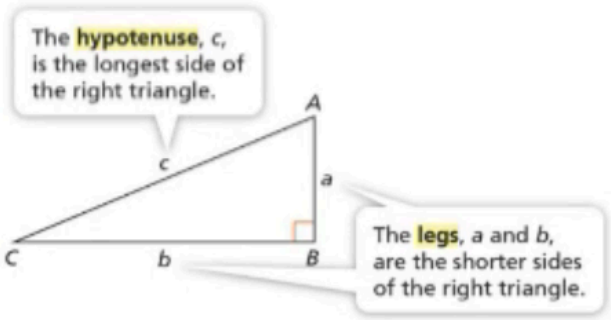
△*ABC* is a right triangle with side lengths *a*, *b*, and *c*. Construct a logical argument to show that $a^2 + b^2 = c^2$.

Construct Arguments When you think logically and use definitions, properties, and given facts to construct an argument, you are developing a mathematical proof.



Try It!

A right triangle has side lengths 15 centimeters, 25 centimeters, and 20 centimeters. How can you use the Pythagorean Theorem to write an equation that describes how the side lengths are related?

$a^2 + b^2 = c^2$

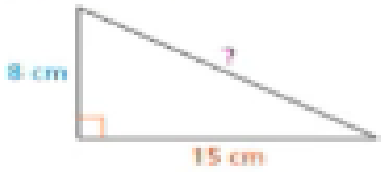
$\square^2 + \square^2 = \square^2$

$\square + \square = \square$

Convince Me! How do you know that the geometric proof of the Pythagorean Theorem shown above can be applied to all right triangles?

EXAMPLE 2 Use the Pythagorean Theorem to Find the Length of the Hypotenuse

What is the length of the hypotenuse of the right triangle?



Use the Pythagorean Theorem.

(leg)^2 + (leg)^2 = (hypotenuse)^2

$a^2 + b^2 = c^2$

$^2 + \quad ^2 = c^2$

$\quad + \quad = c^2$

$\quad\quad = c^2$

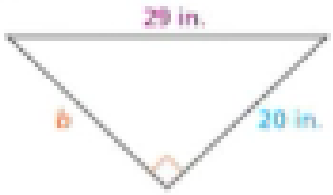
$\sqrt{\quad} = \sqrt{c^2}$

$\quad\quad = c$

The length of the hypotenuse is centimeters.

EXAMPLE 3 Use the Pythagorean Theorem to Find the Length of a Leg

Mara is repairing the trim on one side of a display case sketched at the right. She has a piece of trim that is 20 inches long. Does Mara have enough trim to repair the display case?



The display case is in the shape of a right triangle. Use the Pythagorean Theorem to find the missing side length.

$a^2 + b^2 = c^2$

$^2 + b^2 = ^2$ Substitute the given information.

$+ b^2 =$

$+ b^2 - \quad =$

$b^2 =$

$\sqrt{b^2} = \sqrt{\quad}$

$b =$

Mara needs a inch piece of trim, so she does not have enough trim to repair the display case.

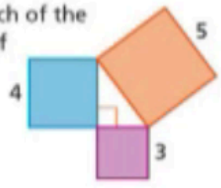
Try It!

A right triangle has a hypotenuse length of 32 meters. It has one leg with a length of 18 meters. What is the length of the other leg? Express your answer as a square root.

Do You Understand?

1. **Essential Question** How does the Pythagorean Theorem relate the side lengths of a right triangle?

2. **Use Structure** A side of each of the three squares forms a side of a right triangle.

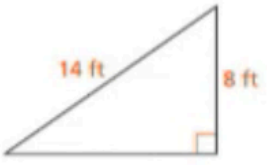


Would any three squares form the sides of a right triangle? Explain.

Do You Know How?

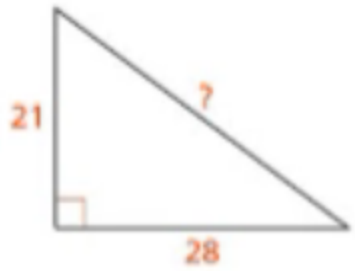
4. A right triangle has leg lengths of 4 inches and 5 inches. What is the length of the hypotenuse? Write the answer as a square root and round to the nearest tenth of an inch.

5. Find the missing side length to the nearest tenth of a foot.

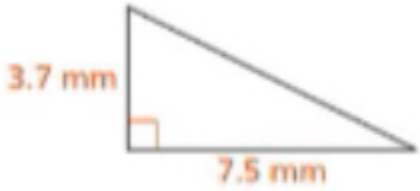


SEE NEXT PAGE FOR #3 AND #6.

3. **Construct Arguments** Xavier said the missing length is about 18.5 units. Without calculating, how can you tell that Xavier solved incorrectly?



6. Find the missing side length to the nearest tenth of a millimeter.



Use for Extra Class Practice:

The Pythagorean Theorem describes the relationship between the lengths of sides for any _____.

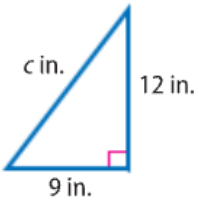
The two shorter sides of the right triangle are called _____.

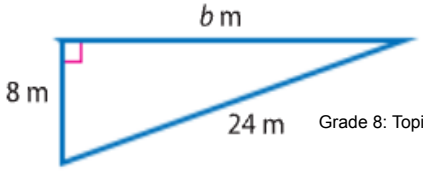
The longest side of a right triangle is called the _____ and is across from the right angle.

$$a^2 + b^2 = c^2$$

You can use the theorem to find an unknown side when you know the other two.

A _____ is a logical mathematical argument in which every fact is supported by a _____.





Lesson 2: Understand the Converse of the Pythagorean Theorem

*Goal: Understand and apply the **Converse** of the **Pythagorean Theorem***
*Use the converse of the Pythagorean Theorem to **analyze triangles***

How can you determine whether a triangle is a right triangle?

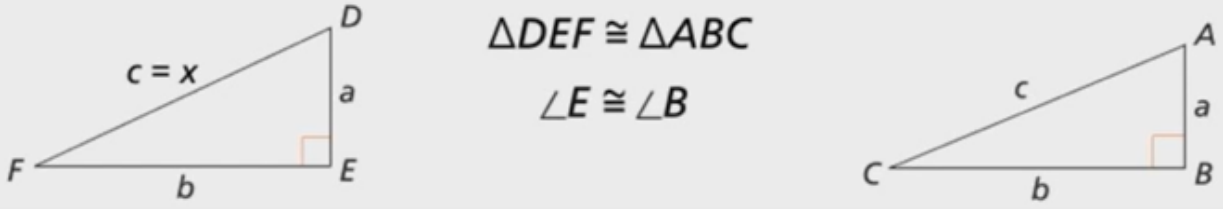
If the length of the three sides of a triangle satisfy the equation

$a^2 + b^2 = c^2$

then the triangle is a right triangle.

This is a proof of the **Converse of the Pythagorean Theorem**.

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle



ESSENTIAL: In order for this to be a right triangle, the Pythagorean Theorem MUST be TRUE!

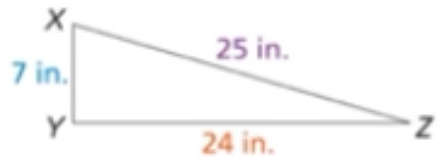
Example:

A triangle has side lengths 4 inches, 5 inches, and 7 inches. Is the triangle a right triangle?

$a^2 + b^2 \stackrel{?}{=} c^2$

YES or NO?

A. Is ΔXYZ a right triangle?



Apply the Converse of the Pythagorean Theorem.

$$a^2 + b^2 \stackrel{?}{=} c^2$$

YES or NO?

B. The side lengths of a triangle are 6 inches, 4.5 inches, and 3.75 inches. Is this triangle a right triangle?

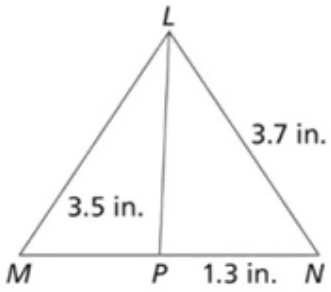
Apply the Converse of the Pythagorean Theorem.

$$a^2 + b^2 \stackrel{?}{=} c^2$$

Yes or No?

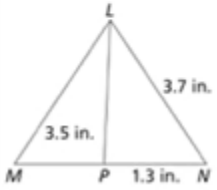
A triangle has side lengths 10 feet, $\sqrt{205}$ feet, and $\sqrt{105}$ feet. Is this a right triangle? Explain.

Rey drew the isosceles triangle *LMN* and the segment *LP*. How can Rey tell whether the segment drawn is the height of the triangle?



What is an isosceles triangle?_____

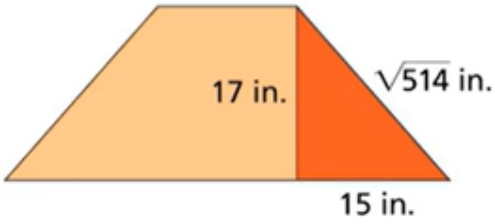
Remember, the corresponding base and height of a triangle are perpendicular. If segment *LP* is the height of Δ*LMN*, then Δ*LPN* is a right triangle.



Use the Converse of the Pythagorean Theorem to determine whether Δ*LPN* is a right triangle.

$a^2 + b^2 \stackrel{?}{=} c^2$

A triangle is inside a trapezoid. Is the triangle a right triangle? Explain.

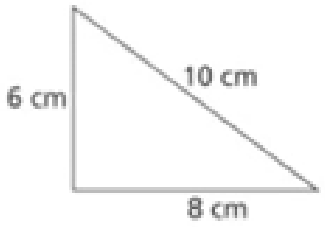


The Converse of the Pythagorean Theorem states that if the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, the triangle is a right triangle.

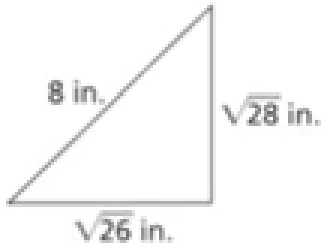
Converse of Pythagorean Theorem
If $a^2 + b^2 = c^2$, then a triangle is a right triangle.

When you are given three side lengths for a triangle, how do you know which length to substitute for a, b, or c in the Pythagorean Theorem?

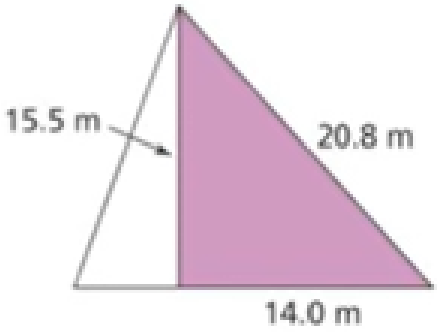
4. Is the triangle a right triangle? Explain.



5. Is the triangle a right triangle? Explain.



6. Is the purple triangle a right triangle? Explain.



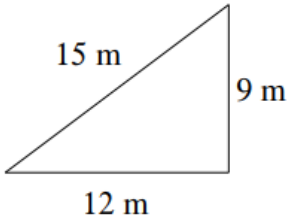
For Extra Practice

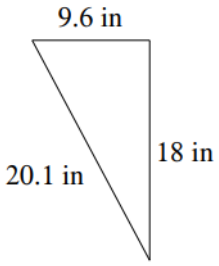
If the three measurements for a triangle work in the Pythagorean Theorem, then they are the measurements of the sides of a _____. This is proof of the _____ of the Theorem.

Remember: The _____ side is the _____ or ____. The 2 _____ sides are the _____ and are the _____ and _____.

Given 3 measurements, determine if they make a right triangle. (*Pythagorean Triple*)

5-12-13 3-4-5 5-10-12



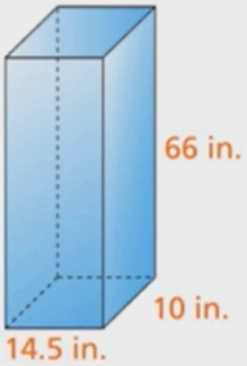


Lesson 3: Apply the Pythagorean Theorem to Solve Problems

*Goal: Use the **Pythagorean Theorem and its Converse** to solve problems*

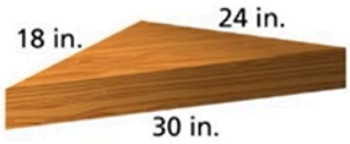
Apply the Pythagorean Theorem to Triangles in Three Dimensions

Alex has a column aquarium with a rectangular base that has a height of 66 inches, a length of 10 inches, and a width of 14.5 inches. What is the longest piece of choya wood that Alex can buy to fit in his tank?



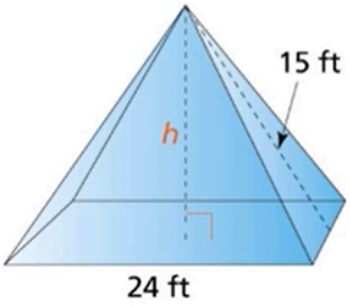
Apply the Converse of the Pythagorean Theorem to Solve Problems

Sandra bought a triangular shelf to hang in the corner of her room. Will this shelf fit in the 90° corner? Explain.



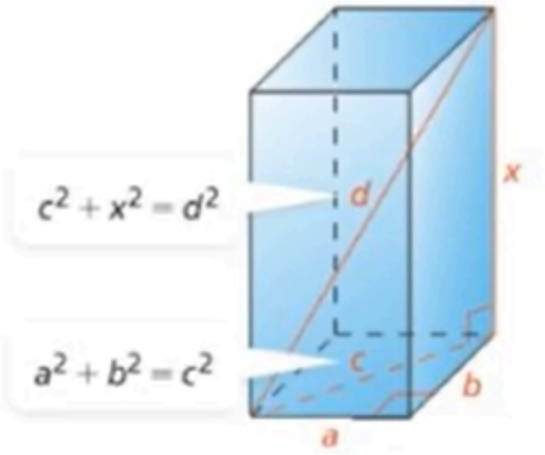
Apply the Converse of the Pythagorean Theorem to Solve Problems Try It

A company wants to rent a tent that has a height of at least 10 feet for an outdoor show. Should they rent the tent shown at the right? Explain.



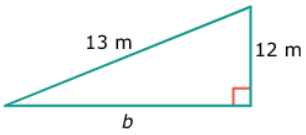
Concept Summary

You can use the Pythagorean Theorem and its converse to solve problems involving right triangles.



Extra Problems for Review

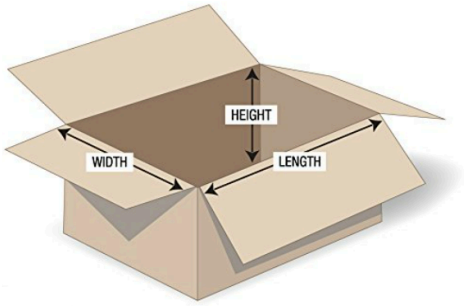
What is the perimeter of the triangle?



To repair a roof that is 4 meters high, Mr. Benton leans a 5-meter ladder against the side of the building. To reach the roof, how far away from the building should he place the base of the ladder?

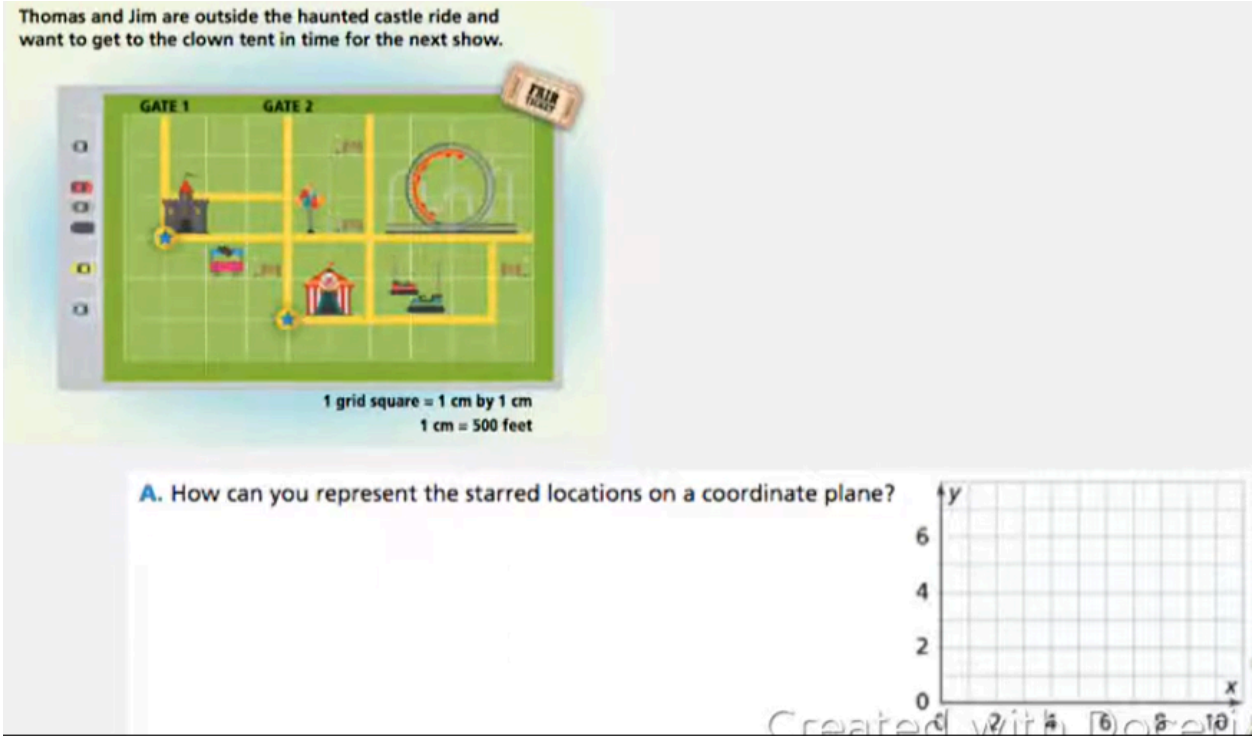
Castroville is 8 kilometers due north of the airport, and Hampton is due east of the airport. If the distance between Castroville and Hampton is 17 kilometers, how far is Hampton from the airport?

The standard box from Amazon measures 16 in. long by 12 in. wide by 8 in. high. What is the largest length vacuum extender that can be shipped in this type of box?



Lesson 4: Find Distance in the Coordinate Plane

*Goal: Apply the Pythagorean Theorem to find the **distance** between two points*
*Use the Pythagorean Theorem to find the **perimeter** of a figure*



EXAMPLE 1

Apply the Pythagorean Theorem to Find the Distance Between Two Points

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Thomas says that walking along a straight path from the haunted mansion to the clown tent is the shorter path. How can you use the Pythagorean Theorem to determine whether he is correct?

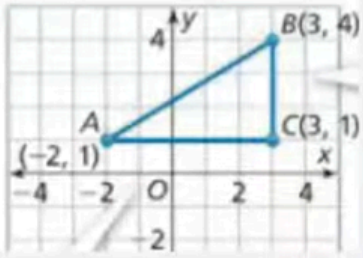
Model with Math How can you use a coordinate plane and the Pythagorean Theorem to represent and find the distance Thomas will walk?

1 grid square = 1 cm by 1 cm
1 cm = 500 feet

EXAMPLE 2

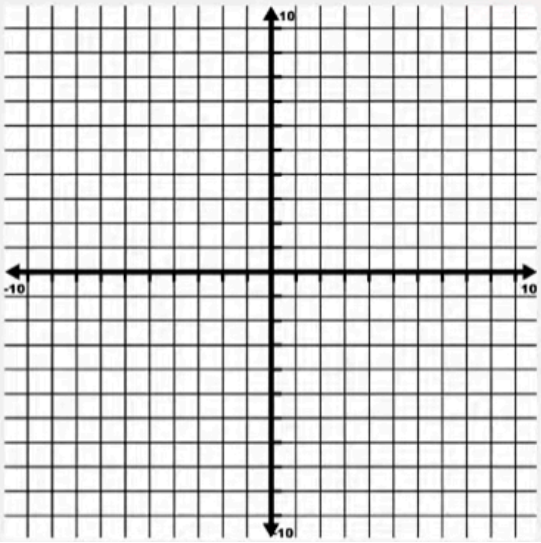
Find the Perimeter of a Figure on a Coordinate Plane

Find the perimeter of ΔABC.



EXAMPLE 3 Use the Pythagorean Theorem to Solve Problems on the Coordinate Plane

Li draws one side of an equilateral triangle with vertices (− 1, 1) and (3, 1) on the coordinate plane. The third vertex is in the first quadrant. What are the coordinates of the third vertex of Li's triangle?

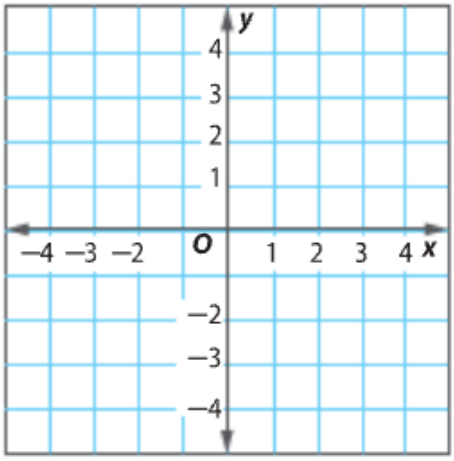


Extra Problems for review

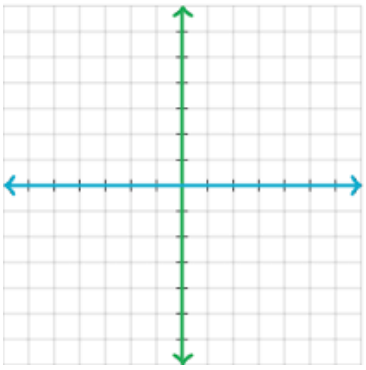
You can use the Pythagorean Theorem to find distance on the coordinate plane.

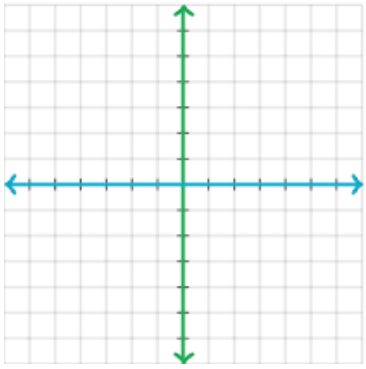
(1,3) and (-2,4)

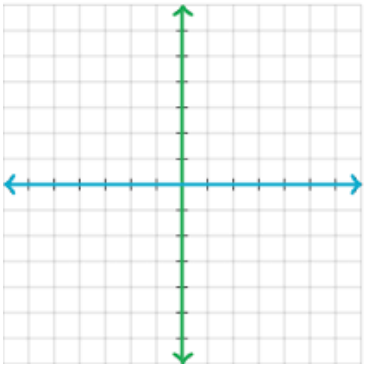
1. Graph 2 ordered pairs
2. Find the _____ distance
3. Find the _____ distance
4. Replace those distances with _____ in formula
5. Solve for _____



1. (4, 5) & (2, 2)
2. (-3, 4) & (1, 3)
3. (6, 2) & (1, 0)





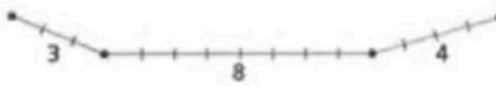


Lesson 5: Draw Triangles with Given Sides

Goal: ***Sketch and construct*** *triangles with given conditions*
Determine the number *of triangles that can be formed given side lengths and angle measures*

1. Can you draw a triangle with side lengths 3, 4, and 8 units? There are different ways to model the situation and investigate.

   A. Use the longest side of your model as the possible base. Can you draw a triangle? Why or why not?

   _____

   B. Repeat Part A using the shortest segment as the possible base. Can you draw a triangle? Why or why not?

   _____

   C. Complete the statements describing the relationship among the three side lengths that do **not** form a triangle. Use *less than, equal to,* or *greater than.*

      The sum of the lengths of the two shorter sides is _____ the length of the longer side.

The _____ _____ _____ states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

2. In Parts A - C, let *a* and *b* be the shorter lengths and *c* be the longest lengths. Compare $a+b$ to c to determine if a triangle can be made. Write <, =, or >.

   A. Nia wants to make a triangular picture frame from strips of wood that are 9cm, 11cm, and 15cm long. _____

   Since the sum of the lengths of two shorter strips is _____ the length of the of the longest strip, Nia **can / can not** make a triangle.

   B. Gerard has pieces of string 6 inches, 5 inches, and 11 inches in length that he plans to use as a border for a collage. 6 + 5 _____ 11. Since the sum of the lengths of two shorter pieces is _____ the length of the longest piece, Gerard **can / can not** make a triangle.

C. Olivia gives her niece leftover pieces of ribbon from her art supplies. They are 12 inches, 10 inches, and 24 inches long. 12 + 10 _____ 24. Since the sum of the lengths of two shorter pieces of ribbon is _____ the length of the longest, Olivia's niece **can / can not** make a triangle.

D. Amil is making a bamboo picture frame. Given the side lengths shown for the first two sides, what is one possible side length that will form a triangular picture frame?

5 + 8 = _____, so one side length that will make a triangle is _____ inches.

Check Understanding

1. Max has three pieces of oak trim that are 7 inches, 11 inches, and 18 inches long. He wants to use them to make a triangular base for a candle holder. Will the pieces make a triangle? Explain your answer.

_____

_____

2. Bella is making a sculpture. She has pieces of copper pipe that are 4cm long and 13 cm long. What is a possible third length of copper wire that will make a triangle. Justify your answer.

_____

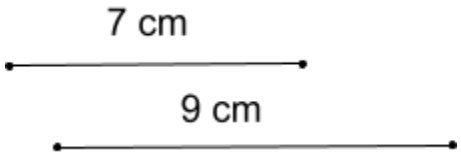
Extra problems for practice

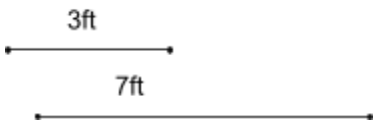
The Triangle Inequality Theorem states that the _____ of the lengths of 2 sides of a triangle is _____ than the length of the third side.

- ☐ A smaller side (minimum): must be larger than the _____ between the 2 sides

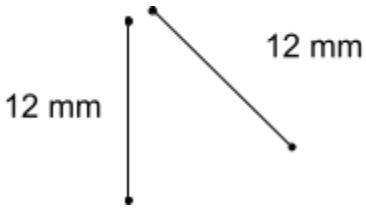
- ☐ A larger side (maximum): must be smaller than the _____ of the 2 sides

What are the possible lengths of the third side of the triangle?





What are the possible lengths of the third side of the triangle?

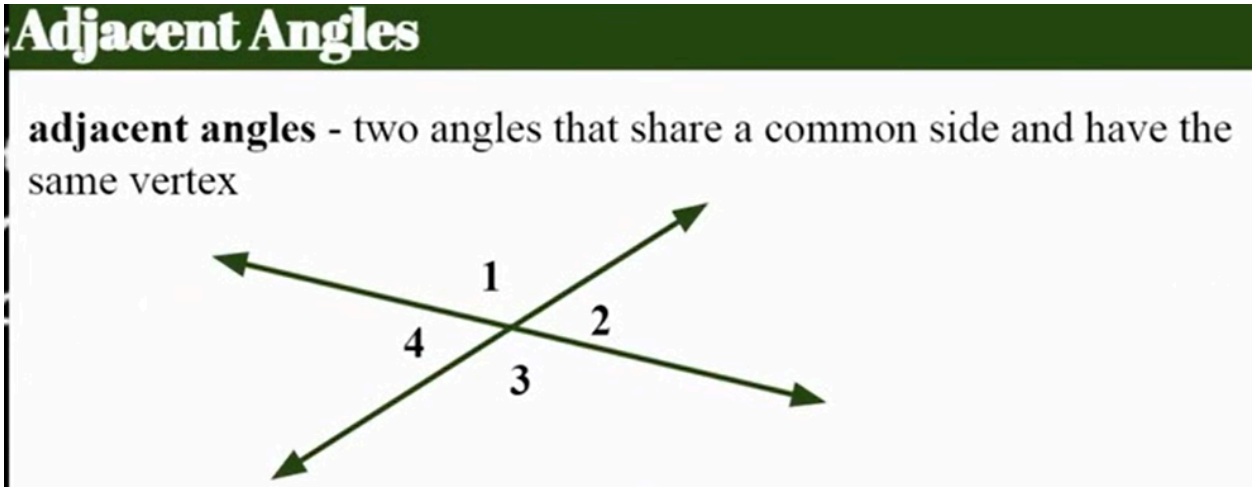


Lesson 6: Solve Problems Using Angles

*Goal: Find the **measure of angles** using angle relationships*
Recognize** the relationship between different angles formed by **intersecting lines

Pre-6-6 Skills - Adjacent and Vertical Angles:

1. *Congruent means _____ size and _____shape.*
2. *Complementary angles ALWAYS add up to _____degrees.*
3. *Supplementary angles ALWAYS add up to _____degrees. _____ degrees is ALWAYS a _____ line.*

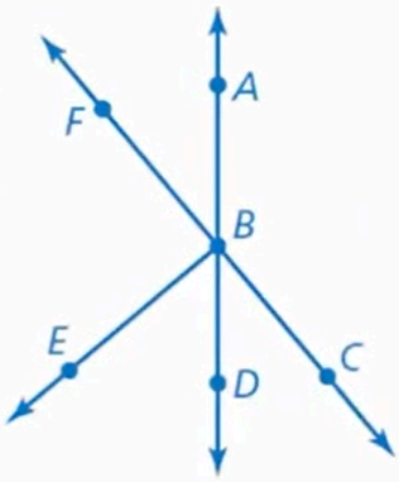


Write the list of adjacent angles here:

Write the list of non-adjacent angles here:

Your Turn

Name two pairs of adjacent angles.

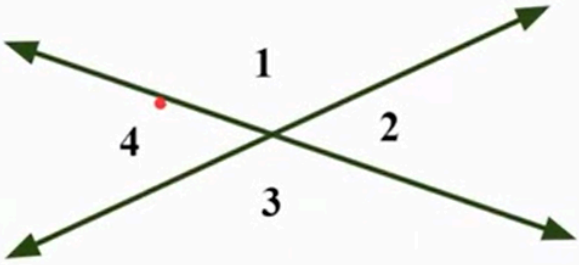


∠ _____ & ∠ _____

∠ _____ & ∠ _____

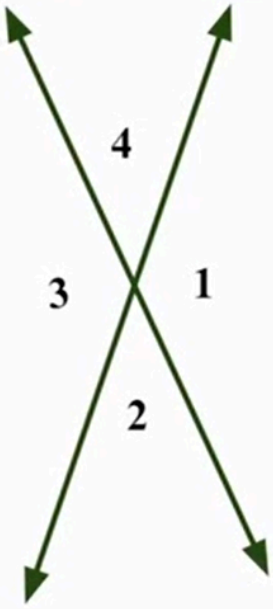
Vertical Angles

vertical angles - two angles that are opposite each other formed by the intersection of two lines.



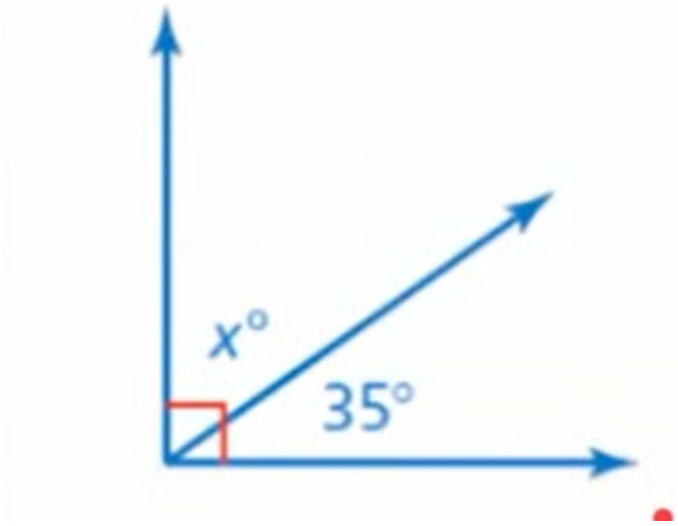
Your Turn

Name two pairs of vertical angles.



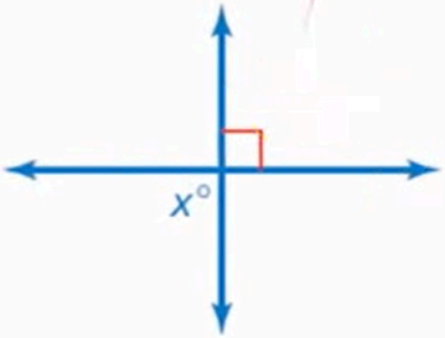
∠ _____ & ∠ _____

∠ _____ & ∠ _____



Finding Vertical Angle Measures

Find the value of x to determine the measure of the missing angle.



These are _____ angles. _____ angles are congruent and have the _____ measure.

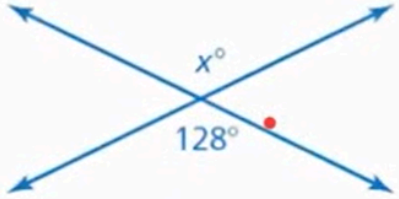
$x =$ _____

\

These are angles that together form a angle. A angle has a measure of degrees.

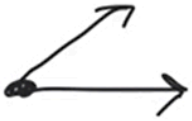
Your Turn

Find the value of x to determine the measure of the missing angle.



6-6 Video Notes:

Explore It!



List all the pairs of angles that share a ray.

Suppose the measure of ∠1 increases. What happens to the size of ∠2? ∠3?

How does the sum of the measures of ∠1 and ∠2 change when one ski moves? Explain.

Types of Angles

Vertical Angles - angles that are _____ each other. They have _____ measures.

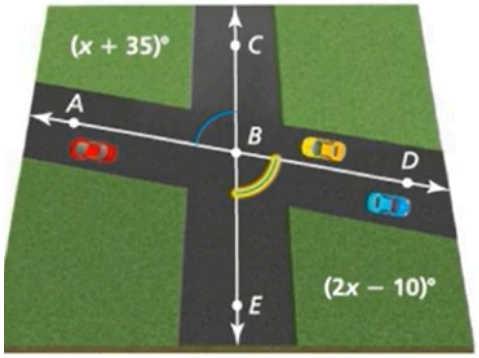
Adjacent Angles - angles that are non-overlapping and share a ray. (___ by ___)

Complementary Angles - angles who have a sum of ___ degrees.

Supplementary Angles - angles who have a sum of ___ degrees.

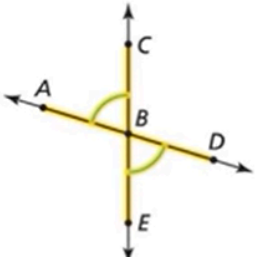
Solve Problems Involving Adjacent and Vertical Angles

A skewed intersection has two roads that intersect at more than 20 degrees away from 90°. Determine whether the road intersection shown is skewed by finding the measures of $\angle ABC$ and $\angle DBE$.

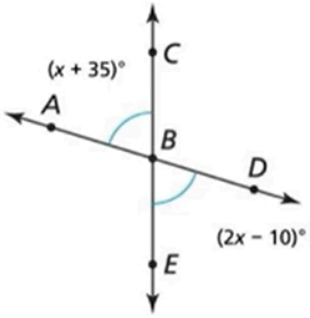


Solve Problems Involving Adjacent and Vertical Angles

Vertical Angles - angles that are opposite each other and are equal in measurement.



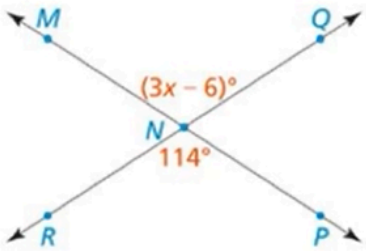
Find the measure of an angle in the intersection.



Solve Problems Involving Adjacent and Vertical Angles

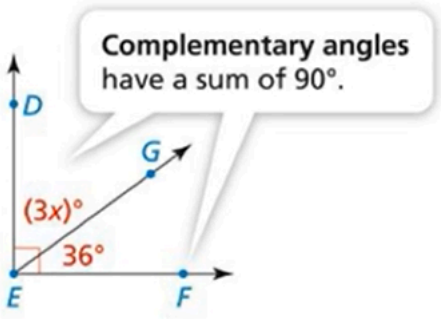
$\angle MNQ$ and $\angle PNR$ are vertical angles.

Find the value of x?



Solve Problems Involving Complementary and Supplementary Angles

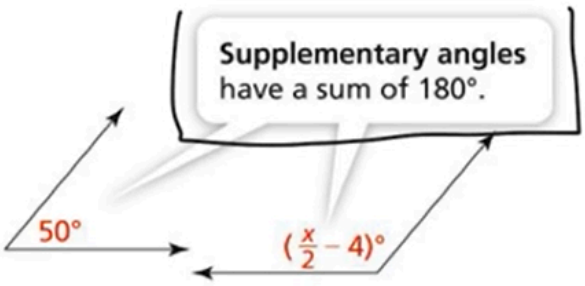
Ray EG splits right angles DEF into two angles. Find the value of x.



$$m\angle DEG + m\angle GEF = 90$$

Solve Problems Involving Complementary and Supplementary Angles

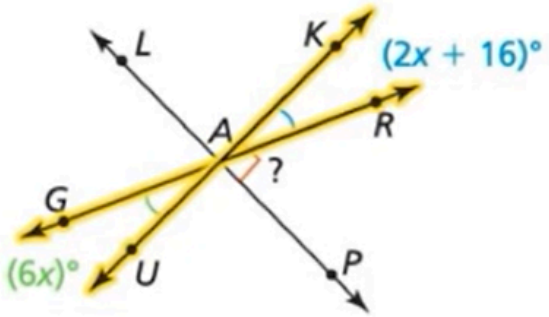
The two angles shown are supplementary angles. Find the value of x.



Find the Measure of an Unknown Angle

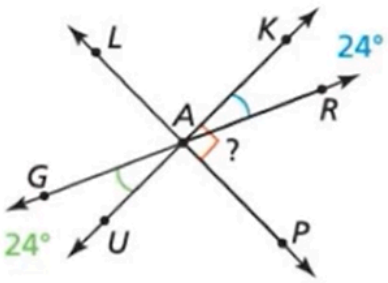
Find the measure of ∠*PAR*.

Use vertical angles to find the value of x.



Find the Measure of an Unknown Angle

Use complementary angles to find the measure of angle PAR.



Find the Measure of an Unknown Angle

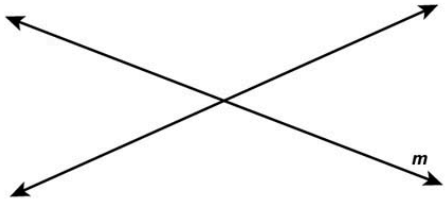
$m\angle 1$ is 4 times $m\angle 2$. $\angle 1$ and $\angle 2$ are complementary. $\angle 1$ and $\angle 3$ are vertical angles. $\angle 3$ and $\angle 4$ are supplementary. What are the measures of the four angles?

Extra Practice Questions:

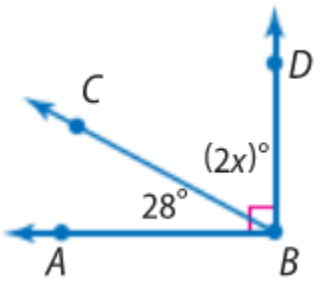
_____ angles have the same measurement.

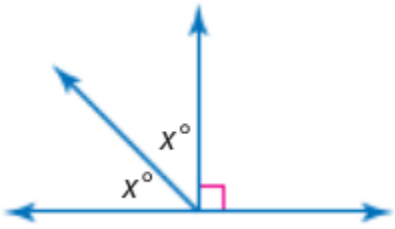
_____ angles that do not overlap but share a common side

_____ angles that are opposite each other and share a common vertex. These angles are always equal



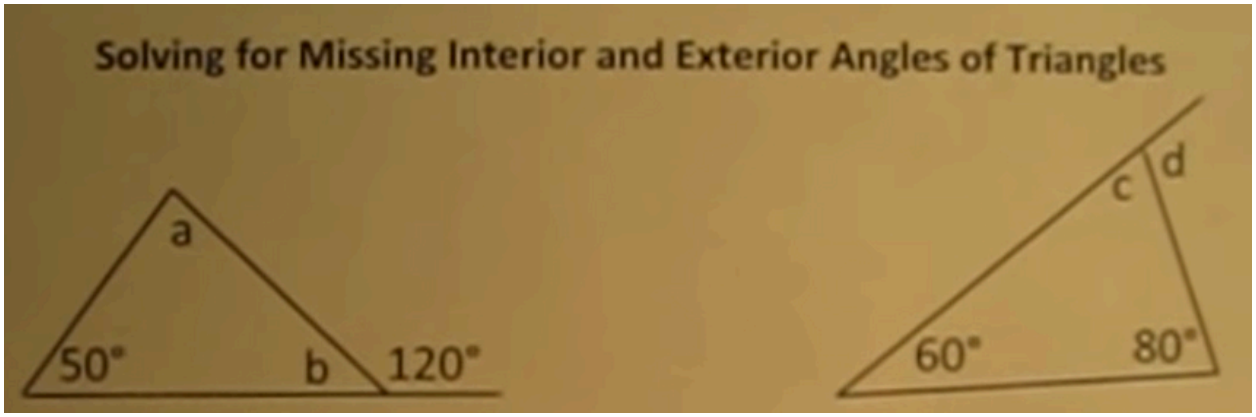
Type of Angle	Description	Example
Complementary		
Supplementary		





Lesson 7: Interior and Exterior Angles of Triangles

Goal: ***Determine unknown measures*** *of angles of triangles*
Write and solve equations *to find angle measures*



Fill in the blanks, please!

An _____ of a triangle is _____ to its adjacent _____.

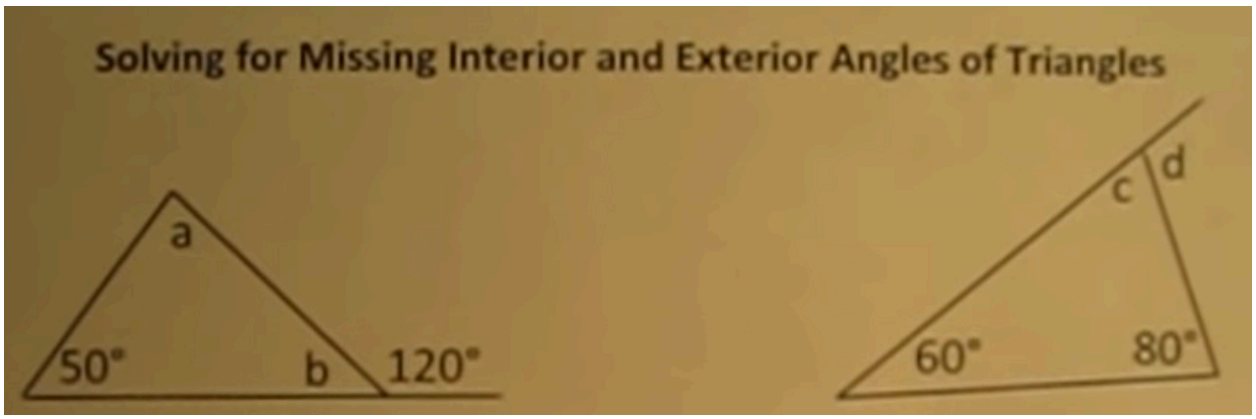
The _____ of the _____ of any triangle is _____.

The measure of an _____ of a triangle is equal to the sum of the measures of the two _____.

The _____ of the interior angles of any triangle is _____.

The _____ of a triangle is _____ to its adjacent _____.

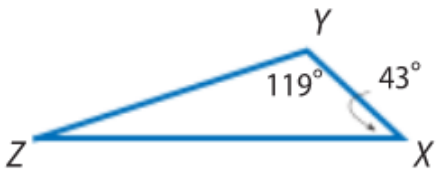
Next show the work to solve here:



More area to show the work to solve.

Extra Practice Problems

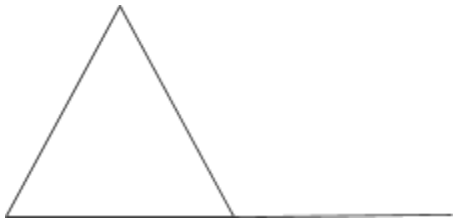
The Triangle Sum Theorem states that the total of all 3 angles in a triangle is _____ degrees

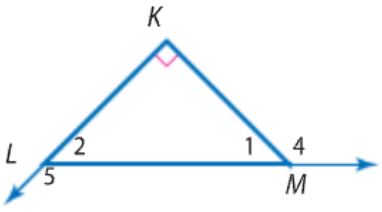


Find the m< Z

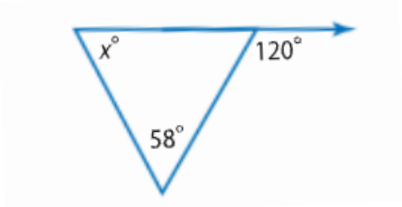
In Triangle ABC, the m<A = 25° and m<B = 108° What is the m<C?

_____ angles are formed by the extension of a side of a triangle. The measure of an exterior angle of a triangle is equal to the _____ of its two remote _____ angles.





If the m<4 = 135°, find m<2

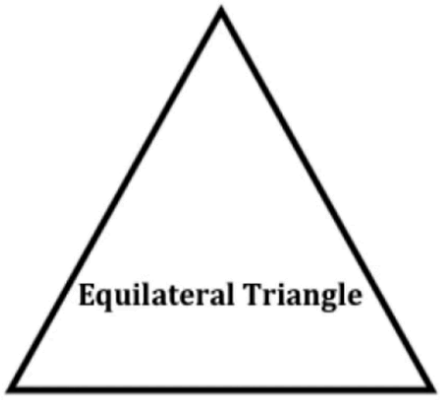


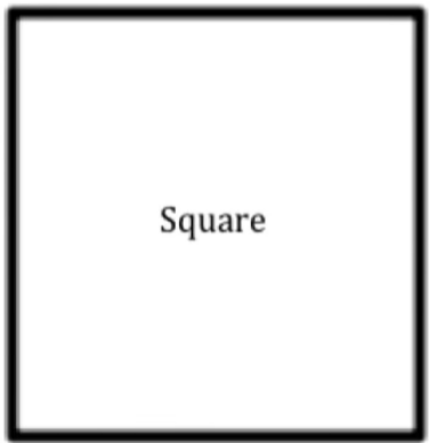
Find m<x

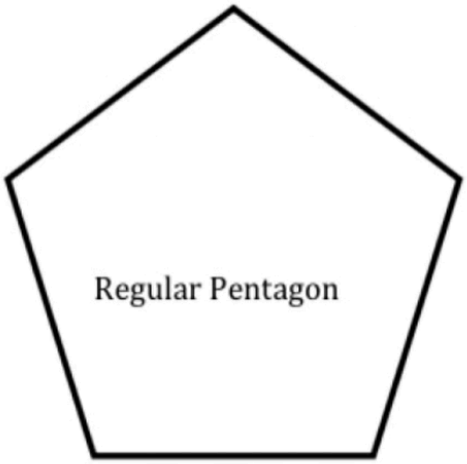
Lesson 8: Interior Angles of Regular Polygons

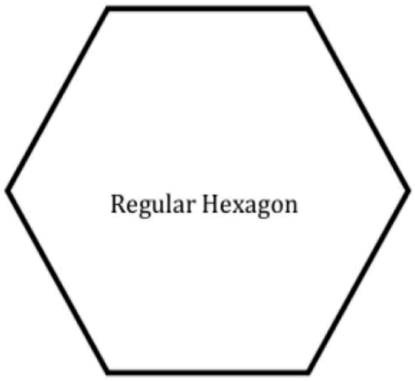
Goal: ***Develop and apply the formula*** *for the* ***sum of interior angles*** *of* ***regular polygons***

Interior Angles of Regular Polygons

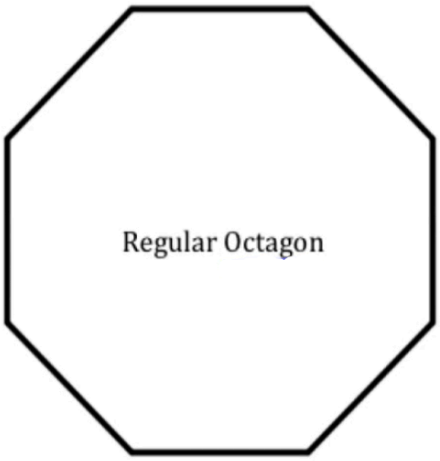





Regular Pentagon


Regular Hexagon


Regular Heptagon


Regular Octagon


Regular Nonagon

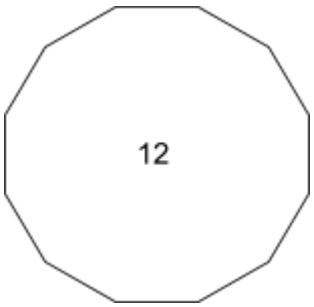

Regular Decagon

Extra Practice Problems:

Polygon: _____

Regular polygon: _____

Number of Sides **n**	Number of Triangles	Sum of Angles $S= (n-2) \cdot 180°$



30- gon

27- gon